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Joint Remote Preparation of a Multipartite GHZ-class State

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Abstract For two parties sharing the original state, a scheme for remote preparation of the two-particle entangled state by three partial two-particle entangled states as the quantum channel is presented, and then directly generalize the scheme for remotely preparing a multipartite GHZ-class state for M senders. It is shown that the receiver can obtain the unknown state with certain probability under the condition that only and only if all the senders collaborate with each other. The N-particle projective measurement and the von Neumann measurement are needed in our scheme. The probability of the successful remote state preparation and classical communication cost are calculated.

Keywords Remote state preparation · N-particle entangled GHZ state · Classical information

1 Introduction

With the development of quantum information, entangled states have been recognized as special physical resources and have served as quantum channels to accomplish various intriguing tasks in quantum information processing. On the other hand, No-cloning theorem forbids a perfect copy of an arbitrary unknown quantum state. How to interchange different resources has ever been a concernment task in quantum communications. The quantum teleportation, firstly proposed by Bennett et al. [1], given a way to transmit an unknown quantum state from a sender to a spatially distant receiver with the help of some classical message and EPR channels without ever physically sending the qubit. Since the sender needs to perform a Bell-state measurement (BM) on his particle and BM has four possible results, two bits of forward classical communication and one ebit of entanglement per teleported qubit are both

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necessary. However, in this process, the sender only owns the original state but has no information about it. If the original state is completely known by Alice, what will happens? To answer this question, Lo [2], Pati [3] and Bennett et al. [4] have presented another interesting novel way to transmit pure known entangled state by using a prior shared entanglement and some classical information. This communication protocol is called remote state preparation (RSP). In RSP, the sender can help a remote receiver to prepare a state by von Neumann Measurement (vNM), and the classical communication cost is reduced to just one cbit. Although RSP protocol is more economical than teleportation for some special ensembles [3], but for general states and a large amount of previously shared entanglement [2, 4], the asymptotic classical communication cost is one bit per qubit in RSP which required as much classical communication cost as quantum teleportation. Since the sender provides with complete classical knowledge on the qubit state and performs vNM on his particles, the required resource can be traded-off between classical communication cost and entanglement cost in RSP, which are not possible in teleportation process. Up to now, RSP has attracted many attentions due to its important applications in quantum communications, more RSP schemes are proposed, such as: low-entanglement RSP [5], higher-dimension RSP [6], optimal RSP [7], oblivious RSP [8], generalized RSP [9], faithful RSP [10], continuous variable RSP [11, 12], and so on. At the same time, some RSP schemes have already been experimentally implemented [13–15]. The other RSP schemes were presented by some authors [16–26].

Let us consider the following scenario. $Alice_1, Alice_2, \ldots, Alice_N$ share a quantum state, if they cooperate with each other, is it possible to help a distant receiver Bob to prepare the original state? How can they do this? By using N-particle GHZ state as the quantum channel, Xia et al. [24] proposed a MRSP scheme for multiparty collaboration with each other for remote preparation of a known quantum state to a distant party. Very recently, two different new quantum protocols [25] to remotely prepare a single-qubit state whose full classical knowledge is split among two distant parties has been proposed. The authors have shown that a third remote party is able to reconstruct the desired state if each other two party performs a vNM. However, it only considers the question of how to remote preparation the single-particle state, furthermore, the participant number is only three. In this letter, we are interested in remotely preparing a multipartite GHZ-class state with more senders. The quantum channels are constructed by N pairs non-maximally EPR states. We will show that remote preparation of the original entangled state can be realized by means of N-particle projective measurement and vNM.

The organization of this paper is as follows. In Sect. 2, using three partial entangled two-particle states quantum channel, we propose a scheme for remotely preparing an unknown two-particle entangled state with three parties. In Sect. 3, we present a scheme of remote preparation of a multipartite GHZ-class state with multiparty based on entangled swapping. Conclusions are given in Sect. 4.

2 Joint Remote Preparation of Two-Particle Entangled State

For simplicity, we firstly present a scheme for remote preparation of two-particle entangled state between three parties. Suppose there are three participants, $Alice_1$, $Alice_2$ and Bob. Here, $Alice_1$ and $Alice_2$ sharing the original state, and they wish to help Bob remotely prepare a two-particle entangled state

$$|\phi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{1}$$

where α is a real number and β is a complex number and $|\alpha|^2+|\beta|^2=1$, $\alpha=\alpha_1\alpha_2$ and $\beta=\beta_1\beta_2$, α_1 and α_2 are real while β_1 and β_2 are complexes, respectively. Suppose that $Alice_1$ and $Alice_2$ know the original state $|\phi\rangle$ partly, that is, $Alice_1$ knows α_1 and β_1, $Alice_2$ knows α_2 and β_2. The quantum channel which shared by $Alice_1$, $Alice_2$ and Bob are three pairs non-maximally EPR state

$$\begin{aligned}
&|\varphi\rangle_{12}=x_1|00\rangle_{12}+y_1|11\rangle_{12}, \qquad |\varphi\rangle_{34}=x_2|00\rangle_{34}+y_2|11\rangle_{34},\\
&|\varphi\rangle_{56}=x_3|00\rangle_{56}+y_3|11\rangle_{56},
\end{aligned} \tag{2}$$

where the coefficients are all nonzero real numbers and satisfy $|x_i|^2+|y_i|^2=1$ and $|x_i|\geq|y_i|$ $(i=1,2,3)$, respectively. Suppose particles 1, 3 and 5 belong to $Alice_1$, $Alice_2$ belongs the particle 2, the particles 4 and 6 belong to Bob. In order to help Bob to remotely prepare the original state, $Alice_1$ should perform a three-projective measurement on particles 1, 3 and 5 in a set of mutually orthogonal basis vectors,

$$\begin{aligned}
|\chi^0\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\alpha_1|000\rangle+\beta_1|111\rangle), & |\chi_\perp^0\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\beta_1^*|000\rangle-\alpha_1|111\rangle),\\
|\chi^1\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\alpha_1|001\rangle+\beta_1|110\rangle), & |\chi_\perp^1\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\beta_1^*|001\rangle-\alpha_1|110\rangle),\\
|\chi^2\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\alpha_1|010\rangle+\beta_1|101\rangle), & |\chi_\perp^2\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\beta_1^*|010\rangle-\alpha_1|101\rangle),\\
|\chi^3\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\alpha_1|011\rangle+\beta_1|100\rangle), & |\chi_\perp^3\rangle&=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}(\beta_1^*|011\rangle-\alpha_1|100\rangle).
\end{aligned} \tag{3}$$

The above eight non-maximally entangled states are related to the computation basis vectors $\{|000\rangle,|111\rangle,|001\rangle,|110\rangle,|010\rangle,|101\rangle,|011\rangle,|100\rangle\}$, and form a complete orthogonal basis in a basis of eight-dimensional Hilbert space. Since the coefficients are not satisfies to [25] $\alpha_1^2+|\beta_1|^2=1$ and $\alpha_2^2+|\beta_2|^2=1$, the above measurement basis must be normalized. Thus, three non-maximally EPR pairs shared previously by $Alice_1$, $Alice_2$ and Bob can be written as

$$\begin{aligned}
|\psi\rangle&=|\varphi\rangle_{12}\otimes|\varphi\rangle_{34}\otimes|\varphi\rangle_{56}\\
&=F_1[|\chi^0\rangle_{135}(\alpha_1x_1x_2x_3|000\rangle+\beta_1^*y_1y_2y_3|111\rangle)_{246}\\
&\quad+|\chi_\perp^0\rangle_{135}(\beta_1x_1x_2x_3|000\rangle-\alpha_1y_1y_2y_3|111\rangle)_{246}\\
&\quad+|\chi^1\rangle_{135}(\alpha_1x_1x_2y_3|001\rangle+\beta_1^*y_1y_2x_3|110\rangle)_{246}\\
&\quad+|\chi_\perp^1\rangle_{135}(\beta_1x_1x_2y_3|001\rangle-\alpha_1y_1y_2x_3|110\rangle)_{246}\\
&\quad+|\chi^2\rangle_{135}(\alpha_1x_1y_2x_3|010\rangle+\beta_1^*y_1x_2y_3|101\rangle)_{246}\\
&\quad+|\chi_\perp^2\rangle_{135}(\beta_1x_1y_2x_3|010\rangle-\alpha_1y_1x_2y_3|101\rangle)_{246}\\
&\quad+|\chi^3\rangle_{135}(\alpha_1x_1y_2y_3|011\rangle+\beta_1^*y_1x_2x_3|100\rangle)_{246}\\
&\quad+|\chi_\perp^3\rangle_{135}(\beta_1x_1y_2y_3|011\rangle-\alpha_1y_1x_2x_3|100\rangle)_{246}],
\end{aligned} \tag{4}$$

where $F_1=\frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}$. Obviously, after $Alice_1$'s measurement, the initial state can be projected onto the generalized measurement basis with certain probability, and a process of entanglement swapping [27] happens. For example, if $Alice_1$'s measurement outcome is $|\chi_\perp^1\rangle_{135}$, with probability $F_1^2[(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2]$. In this case, the state of particles 2, 4 and 6 will collapse into

$$|\mu\rangle_{246}=\frac{1}{\sqrt{(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2}}(\beta_1x_1x_2y_3|001\rangle-\alpha_1y_1y_2x_3|110\rangle)_{246}. \quad (5)$$

Then she informs Bob of the measurement result via classical channel. For help Bob to prepare the original state, $Alice_2$ needs to perform a vNM on her particle 2 in the basis $\{|\lambda\rangle,|\lambda_\perp\rangle\}$, which are related to the computation basis vectors $\{|0\rangle,|1\rangle\}$, is given by

$$|\lambda\rangle_2=\frac{1}{\sqrt{\alpha_2^2+|\beta_2|^2}}(\alpha_2|0\rangle_2+\beta_2|1\rangle_2), \qquad |\lambda_\perp\rangle_2=\frac{1}{\sqrt{\alpha_2^2+|\beta_2|^2}}(\beta_2^*|0\rangle_2-\alpha_2|1\rangle_2). \quad (6)$$

In this way, we can get

$$\begin{aligned}|\mu'\rangle_{246}=&\frac{1}{\sqrt{(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2}}[F_2|\lambda\rangle_2(\alpha_2\beta_1x_1x_2y_3|01\rangle-\beta_2^*\alpha_1y_1y_2x_3|10\rangle)_{46})\\&+F_2|\lambda_\perp\rangle_2(\beta_1\beta_2x_1x_2y_3|01\rangle+\alpha_1\alpha_2y_1y_2x_3|10\rangle)_{46}], \end{aligned}\quad (7)$$

where $F_2=\frac{1}{\sqrt{\alpha_2^2+|\beta_2|^2}}$. After performing vNM on particle 2, the state of particle 2 can be projected onto the generalized measurement basis. If $Alice_2$'s result is $|\lambda\rangle_2$, Bob will obtain the state

$$\frac{1}{\sqrt{(\alpha_2\beta_1x_1x_2y_3)^2+(\beta_2^*\alpha_1y_1y_2x_3)^2}}(\alpha_2\beta_1x_1x_2y_3|01\rangle-\beta_2^*\alpha_1y_1y_2x_3|10\rangle)_{46}). \quad (8)$$

Since Bob has no knowledge of these states, he cannot find any unitary operation to convert it into the original state $|\phi\rangle$, the RSP fails. If $Alice_2$'s result is $|\lambda_\perp\rangle_2$, according to $Alice_2$'s measurement result, Bob knows the state of his particles 4 and 6, as shown by (7), will collapse into

$$\begin{aligned}|\nu\rangle_{26}&=\frac{1}{\sqrt{(\alpha_1\alpha_2y_1y_2x_3)^2+(\beta_1\beta_2x_1x_2y_3)^2}}(\alpha_1\alpha_2y_1y_2x_3|10\rangle+\beta_1\beta_2x_1x_2y_3|01\rangle)_{46}\\&=\frac{1}{\sqrt{(\alpha y_1y_2x_3)^2+(\beta x_1x_2y_3)^2}}(\alpha y_1y_2x_3|10\rangle+\beta x_1x_2y_3|01\rangle)_{46}.\end{aligned} \quad (9)$$

Now, let us see how the original state $|\phi\rangle$ can be prepared from the state in above equation. Firstly, Bob needs to establish a correspondence that the coefficients α and β can correspond to $|00\rangle$ and $|11\rangle$, respectively. Hence, Bob carries out the unitary transformation $U_1=(\sigma_x)_4\otimes(I)_6$ on his particles 4 and 6, where σ_x and I are the Pauli operator and identity operator. This unitary operation will transform the state $|\nu\rangle_{26}$ into

$$U_1|\nu\rangle_{26}=\frac{1}{\sqrt{(\alpha y_1y_2x_3)^2+(\beta x_1x_2y_3)^2}}(\alpha y_1y_2x_3|00\rangle+\beta x_1x_2y_3|11\rangle)_{46}. \quad (10)$$

secondly, Bob introduces an auxiliary two-level particle C with the initial state $|0\rangle_C$ and performs another unitary operation U_2 on particles 4, 6 and C under the basis $\{|00\rangle_{46}|0\rangle_C, |11\rangle_{46}|0\rangle_C, |00\rangle_{46}|1\rangle_C, |11\rangle_{46}|1\rangle_C\}$, which take the form of the following 4×4 matrix [28] $(|x_i| \geq |y_i|)$

$$U_2 = \begin{pmatrix} \frac{y_3}{x_3} & 0 & \sqrt{1-\frac{y_3^2}{x_3^2}} & 0 \\ 0 & \frac{y_1 y_2}{x_1 x_2} & 0 & \sqrt{1-\frac{y_1^2 y_2^2}{x_1^2 x_2^2}} \\ \sqrt{1-\frac{y_3^2}{x_3^2}} & 0 & -\frac{y_3}{x_3} & 0 \\ 0 & \sqrt{1-\frac{y_1^2 y_2^2}{x_1^2 x_2^2}} & 0 & -\frac{y_1 y_2}{x_1 x_2} \end{pmatrix}. \tag{11}$$

After Bob's collective unitary operation U_2 on the particles 4, 6 and A, the initial joint state which describe in (10) is transformed into

$$\begin{aligned} U_2 U_1 |\nu\rangle_{26}|0\rangle_C = {} & \frac{1}{\sqrt{(\alpha y_1 y_2 x_3)^2 + (\beta x_1 x_2 y_3)^2}} [y_1 y_2 y_3 (\alpha|00\rangle + \beta|11\rangle)_{46} \otimes |0\rangle_C \\ & + (\sqrt{(y_1 y_2 x_3)^2 - (y_1 y_2 y_3)^2}\alpha|00\rangle_{46} \\ & + \sqrt{(x_1 x_2 y_3)^2 - (y_1 y_2 y_3)^2}\beta|11\rangle_{46}) \otimes |1\rangle_C]. \end{aligned} \tag{12}$$

Finally, Bob measures the state of particle C. If the result $|1\rangle_C$ is measured, the RSP fails. If Bob's measurement result is $|0\rangle_C$, he can reconstruct the original state on his particles and the probability that Bob obtain the result $|0\rangle_C$ is $\frac{y_1^2 y_2^2 y_3^2}{(\alpha y_1 y_2 x_3)^2 + (\beta x_1 x_2 y_3)^2}$. Thus, the probability of successful RSP is

$$\begin{aligned} & F_1^2[(\beta_1 x_1 x_2 y_3)^2 + (\alpha_1 y_1 y_2 x_3)^2] \frac{F_2^2[(\alpha y_1 y_2 x_3)^2 + (\beta x_1 x_2 y_3)^2]}{(\beta_1 x_1 x_2 y_3)^2 + (\alpha_1 y_1 y_2 x_3)^2} \frac{y_1^2 y_2^2 y_3^2}{(\alpha y_1 y_2 x_3)^2 + (\beta x_1 x_2 y_3)^2} \\ & \quad = F_1^2 F_2^2 y_1^2 y_2^2 y_3^2. \end{aligned}$$

In the same way, if $Alice_1$'s result is $|\chi_\perp^0\rangle_{135}$, $|\chi_\perp^2\rangle_{135}$, $|\chi_\perp^3\rangle_{135}$ and $Alice_2$'s measurement is $|\lambda_\perp\rangle_2$, Bob also can obtain the original state with certain probability. We can easily find that the successful probability is also $F_1^2 F_2^2 y_1^2 y_2^2 y_3^2$. Surely, it is also possible for $Alice_1$ to get the state $|\chi^0\rangle_{135}$, $|\chi^1\rangle_{135}$, $|\chi^2\rangle_{135}$, $|\chi^3\rangle_{135}$ from (4). In this case, Bob can't reconstruct the original state on his particles whatsoever $Alice_2$'s measurement result is. Hence, the total probability of successful RSP is $P = 4F_1^2 F_2^2 y_1^2 y_2^2 y_3^2$.

Classical communication cost plays an important role in RSP. In our RSP process, two kinds of classical communication processes are involved [19, 21]. One is that the classical information is sent from $Alice_1$ to the Bob (S_1), the other can been seen as from $Alice_2$ to the receiver Bob (S_2). To the first classical communication process, if $Alice_1$'s results is $|\chi^0\rangle_{135}$, $|\chi^1\rangle_{135}$, $|\chi^2\rangle_{135}$ and $|\chi^3\rangle_{135}$, the RSP fails. Thus, she needn't to tell Bob what result she will obtain. On the other hand, after the three-particle projective measurement, $Alice_1$ can obtain four probabilistic results $|\chi_\perp^0\rangle_{135}$, $|\chi_\perp^1\rangle_{135}$, $|\chi_\perp^2\rangle_{135}$ and $|\chi_\perp^3\rangle_{135}$ with the measurement probabilities $p_0 = F_1^2[(\beta_1 x_1 x_2 x_3)^2 + (\alpha_1 y_1 y_2 y_3)^2]$, $p_1 = F_1^2[(\beta_1 x_1 x_2 y_3)^2 + (\alpha_1 y_1 y_2 x_3)^2]$, $p_2 = F_1^2[(\beta_1 x_1 y_2 x_3)^2 + (\alpha_1 y_1 x_2 y_3)^2]$ and $p_3 = F_1^2[(\beta_1 x_1 y_2 y_3)^2 + (\alpha_1 y_1 x_2 x_3)^2]$, respectively.

Table 1 The probability for $Alice_2$'s measurement results corresponding to the outcome of $Alice_1$'s measurement

$Alice_1$'s result	Probability for the state $\|\lambda\rangle$ (p')	Probability for the state $\|\lambda_\perp\rangle$ (p'')
$\|\chi_\perp^0\rangle_{135}$	$p'_0=\frac{F_2^2[(\alpha_2\beta_1x_1x_2x_3)^2+(\alpha_1\beta_2^*y_1y_2y_3)^2]}{(\beta_1x_1x_2x_3)^2+(\alpha_1y_1y_2y_3)^2}$	$p''_0=\frac{F_2^2[(\alpha y_1y_2y_3)^2+(\beta x_1x_2x_3)^2]}{(\beta_1x_1x_2x_3)^2+(\alpha_1y_1y_2y_3)^2}$
$\|\chi_\perp^1\rangle_{135}$	$p'_1=\frac{F_2^2[(\alpha_2\beta_1x_1x_2y_3)^2+(\alpha_1\beta_2^*y_1y_2x_3)^2]}{(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2}$	$p''_2=\frac{F_2^2[(\alpha y_1y_2x_3)^2+(\beta x_1x_2y_3)^2]}{(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2}$
$\|\chi_\perp^2\rangle_{135}$	$p'_2=\frac{F_2^2[(\alpha_2\beta_1x_1y_2x_3)^2+(\alpha_1\beta_2^*y_1x_2y_3)^2]}{(\beta_1x_1y_2x_3)^2+(\alpha_1y_1x_2y_3)^2}$	$p''_2=\frac{F_2^2[(\alpha y_1x_2y_3)^2+(\beta x_1y_2x_3)^2]}{(\beta_1x_1y_2x_3)^2+(\alpha_1y_1x_2y_3)^2}$
$\|\chi_\perp^3\rangle_{135}$	$p'_2=\frac{F_2^2[(\alpha_2\beta_1x_1y_2y_3)^2+(\alpha_1\beta_2^*y_1x_2x_3)^2]}{(\beta_1x_1y_2y_3)^2+(\alpha_1y_1x_2x_3)^2}$	$p''_2=\frac{F_2^2[(\alpha y_1x_2x_3)^2+(\beta x_1y_2y_3)^2]}{(\beta_1x_1y_2y_3)^2+(\alpha_1y_1x_2x_3)^2}$

Thus, the classical information in this process is

$$
\begin{aligned}
S_1 &= -\sum_{i=0}^{3} p_i \log_2[p_i] \\
&= -F_1^2[(\beta_1x_1x_2x_3)^2+(\alpha_1y_1y_2y_3)^2]\log_2 F_1^2[(\beta_1x_1x_2x_3)^2+(\alpha_1y_1y_2y_3)^2] \\
&\quad - F_1^2[(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2]\log_2 F_1^2[(\beta_1x_1x_2y_3)^2+(\alpha_1y_1y_2x_3)^2] \\
&\quad - F_1^2[(\beta_1x_1y_2x_3)^2+(\alpha_1y_1x_2y_3)^2]\log_2 F_1^2[(\beta_1x_1y_2x_3)^2+(\alpha_1y_1x_2y_3)^2] \\
&\quad - F_1^2[(\beta_1x_1y_2y_3)^2+(\alpha_1y_1x_2x_3)^2]\log_2 F_1^2[(\beta_1x_1y_2y_3)^2+(\alpha_1y_1x_2x_3)^2]. \qquad (13)
\end{aligned}
$$

In the second process, after $Alice_2$'s vNM, there are two possible measurement results, the state $|\lambda\rangle_2$ or $|\lambda_\perp\rangle_2$. Corresponding to $Alice_1$'s measurement outcomes, the probability for each result is listed in Table 1. Similarly, if $Alice_2$'s measurement result is $|\lambda\rangle_2$, the RSP fails. In this situation, $Alice_2$ need not to send any classical bit to Bob. So the total amount of the classical communication required in this process is

$$S_2 = -p_0p''_0\log_2[p''_0] - p_1p''_1\log_2[p''_1] - p_2p''_2\log_2[p''_2] - p_3p''_3\log_2[p''_3]. \qquad (14)$$

Therefor, the total classical communication for this probabilistic remote preparation of an entangled two-qubit state with three parties requires

$$S = S_1 + S_2 = -\sum_{i=0}^{3} p_i\log_2[p_i] - \sum_{i=0}^{3} p_ip''_i\log_2[p''_i], \quad i=i''=0,1,2. \qquad (15)$$

Obviously, the classical communication information is not only depended on the small coefficients of the quantum channel, but also by means of the original state which described in (1). If the quantum channel consists of three EPR pair ($x_1=y_1=\frac{1}{\sqrt{2}}$, $x_2=y_2=\frac{1}{\sqrt{2}}$, $x_3=y_3=\frac{1}{\sqrt{2}}$), the total probability of successful RSP equals $\frac{F_1^2F_2^2}{2}$, and classical information will consume $(1.5+\frac{F_1^2F_2^2}{2}\log_2[\frac{1}{F_1^2F_2^2}])$ bits in total.

3 Joint Remote Preparation of a Multipartite GHZ-class State

Now we generalize our scheme to realize the remote preparation of a multipartite GHZ-class state with M senders. First, we consider two parties sharing the original state. Suppose that the senders $Alice_1$ and $Alice_2$ want to help the receiver Bob prepare remotely a multipartite GHZ-class state

$$|\Lambda\rangle_{12} = \alpha|0\cdots 0\rangle_{12\ldots N} + \beta|1\cdots 1\rangle_{12\ldots N}, \tag{16}$$

where the parameter α is real and β is a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. In addition

$$\alpha = \alpha_1\alpha_2, \qquad \beta = \beta_1\beta_2. \tag{17}$$

As same as the Sect. 2, here, $Alice_1$ knows α_1 and β_1, and $Ailce_2$ knows α_2 and β_2, where α_1 and α_2 are real number, and β_1 and β_2 are complexes. Let assume $Alice_1$, $Alice_2$ and Charlie share $N+1$ pairs of non-maximally entangled states, which are given by

$$|\varphi\rangle_{A_iB_i} = x_i|00\rangle_{A_iB_i} + y_i|00\rangle_{A_iB_i}, \quad i = 1, 2, \ldots, N+1, \tag{18}$$

where x_i and y_i are nonzero real numbers, and satisfy to $x_i^2 + y_i^2 = 1$, respectively. We also assume the particles $A_1, A_2, \ldots, A_{N+1}$ are in $Alice_1$'s possession, particle B_1 belongs to $Alice_2$ while the particles $B_2, B_3, \ldots, B_{N+1}$ belong to Bob. To remotely prepare state which describe in (16), the sender $Alice_1$ carries out a $N+1$-particle projective measurement on the particles $A_1, A_2, \ldots, A_{N+1}$ which are given by

$$\begin{aligned}
|\psi^l\rangle_{A_1A_2\ldots A_{N+1}} &= F_1\left(\alpha_1|0\rangle_{A_1} \otimes \prod_{j=2}^{N+1}|\rho(j)\rangle_{A_j} + \beta_1|1\rangle_{A_1} \otimes \prod_{j=2}^{N+1}|\tilde{\rho}(j)\rangle_{A_j}\right),\\
|\psi_\perp^l\rangle_{A_1A_2\ldots A_{N+1}} &= F_1\left(\beta_1^*|0\rangle_{A_1} \otimes \prod_{j=2}^{N+1}|\rho(j)\rangle_{A_j} - \alpha_1|1\rangle_{A_1} \otimes \prod_{j=2}^{N+1}|\tilde{\rho}(j)\rangle_{A_j}\right),
\end{aligned} \tag{19}$$

where $F_1 = \frac{1}{\sqrt{\alpha_1^2+|\beta_1|^2}}$. Furthermore, $|\rho(j)\rangle_{A_j}$ and $|\tilde{\rho}(j)\rangle_{A_j}$ stand for two orthogonal states of the j-th particle, the symbol $\rho(j)$ is a binary variable $\in \{0, 1\}$ with its complement $\tilde{\rho}(j) = 1 - \rho(j)$. For the sake of simplicity, a decimal numeral l can defined as [29]

$$\begin{aligned}
l &= (\rho(2)\rho(3)\cdots\rho(N+1))_2 = \sum_{j=2}^{N+1}\rho(j)2^{N+1-j}\\
&= \rho(2)2^{N-1} + \rho(3)2^{N-2} + \cdots + \rho(N+1)2^0.
\end{aligned} \tag{20}$$

Thus, an unique representation of l is established with a set of binary number $\rho(2), \rho(3), \ldots, \rho(N+1)$, and we can obtain $l = 0, 1, \ldots, 2^N - 1$. To achieve their goal, $Alice_2$ should perform von Neumann measurement on particle B_1, which is given by

$$|\lambda\rangle_{B_1} = F_2(\alpha_2|0\rangle_2 + \beta_2|1\rangle_2), \qquad |\lambda_\perp\rangle_{B_1} = F_2(\beta_2^*|0\rangle_2 - \alpha_2|1\rangle_2), \tag{21}$$

where $F_2 = \frac{1}{\sqrt{\alpha_2^2+|\beta_2|^2}}$. In this case, the $N+1$ pairs of non-maximally two particle entangled state can be written as

$$\prod_{i=1}^{N+1}|\varphi\rangle_{A_iB_i} = F_1F_2\sum_{l=0}^{2^N-1}[|\Upsilon_1\rangle + |\Upsilon_2\rangle + |\Upsilon_3\rangle + |\Upsilon_4\rangle] \tag{22}$$

where

$$|\Upsilon_1\rangle = |\psi^l\rangle_{A_1A_2\dots A_{N+1}}|\lambda\rangle_{B_1}\Bigg(\alpha_1\alpha_2 x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j} + \beta_1^*\beta_2^* y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j}\Bigg), \quad (23)$$

$$|\Upsilon_2\rangle = |\psi^l\rangle_{A_1A_2\dots A_{N+1}}|\lambda_\perp\rangle_{B_1}\Bigg(\alpha_1\beta_2 x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j} - \alpha_2\beta_1^* y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j}\Bigg), \quad (24)$$

$$|\Upsilon_3\rangle = |\psi_\perp^l\rangle_{A_1A_2\dots A_{N+1}}|\lambda\rangle_{B_1}\Bigg(\alpha_2\beta_1 x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j} - \alpha_1\beta_2^* y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j}\Bigg), \quad (25)$$

$$|\Upsilon_4\rangle = |\psi_\perp^l\rangle_{A_1A_2\dots A_{N+1}}|\lambda_\perp\rangle_{B_1}\Bigg(\beta_1\beta_2 x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j} + \alpha_1\alpha_2 y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j}\Bigg), \quad (26)$$

here, we defined

$$X_{\rho(j)} = \begin{cases} x_j, & \text{if } \rho(j)=0, \\ y_j, & \text{if } \rho(j)=1, \end{cases} \qquad Y_{\tilde\rho(j)} = \begin{cases} y_j, & \text{if } \tilde\rho(j)=1, \\ x_j, & \text{if } \tilde\rho(j)=0. \end{cases} \quad (27)$$

After their measuring, they inform Bob of the measurement result via classical channel. For instance, if $Alice_1$'s measurement outcome is $|\psi_\perp^l\rangle_{A_1A_2\dots A_{N+1}}$ and $Alice_2$'s result is $|\lambda_\perp\rangle_{B_1}$, respectively. In this case, as shown by (26), the N-particle $B_2, B_3, \dots, B_{N+1}$ at Bob's location will be in an entangled state

$$\begin{aligned} |\Delta\rangle_{B_2B_3\dots B_{N+1}} &= \Gamma\Bigg(\beta_1\beta_2 x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j} + \alpha_1\alpha_2 y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j}\Bigg) \\ &= \Gamma\Bigg(\alpha y_1 \otimes \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}|\tilde\rho(j)\rangle_{B_j} + \beta x_1 \otimes \prod_{j=2}^{N+1} X_{\rho(j)}|\rho(j)\rangle_{B_j}\Bigg) \end{aligned} \quad (28)$$

where

$$\Gamma = \frac{1}{\sqrt{|\beta|_1^2|\beta|_2^2 x_1^2 \prod_{j=2}^{N+1} X_{\rho(j)}^2 + \alpha_1^2\alpha_2^2 y_1^2 \prod_{j=2}^{N+1} Y_{\tilde\rho(j)}^2}}. \quad (29)$$

In order to reconstruct the original state which described in (16), some appropriate operations are needed to performed by Bob. Firstly, Bob needs to establish a correspondence such that coefficients α and β can correspond to $|0\cdots0\rangle$ and $|1\cdots1\rangle$, respectively. According to the (28), Bob carries out the unitary transformation $U_1=|0\rangle\langle\tilde{\rho}(j)|+|1\rangle\langle\rho(j)|$ on the particles $\{B_2,B_3,\ldots,B_{(N+1)}\}$, which will transform the state $|\Delta\rangle$ into

$$U_1|\Delta\rangle_{B_2B_3\ldots B_{N+1}}=\Gamma\left(\alpha y_1\prod_{j=2}^{N+1}Y_{\tilde{\rho}(j)}|0\cdots0\rangle_{B_2B_3\ldots B_{N+1}}\right.$$
$$\left.+\beta x_1\prod_{j=2}^{N+1}X_{\rho(j)}|1\cdots1\rangle_{B_2B_3\ldots B_{N+1}}\right). \tag{30}$$

Secondly, Bob introduces an auxiliary two-level particle C with initial state $|0\rangle_C$ and performs another unitary operation U_2 on particles $B_2,B_3,\ldots,B_{N+1}$ under basis $\{|0\cdots0\rangle_{B_2B_3\ldots B_{N+1}}|0\rangle_C,\ |1\cdots1\rangle_{B_2B_3\ldots B_{N+1}}|0\rangle_C,\ |0\cdots0\rangle_{B_2B_3\ldots B_{N+1}}|1\rangle_C,\ |1\cdots1\rangle_{B_2B_3\ldots B_{N+1}}|1\rangle_C\}$. If we assume $Y=y_1\prod_{j=2}^{N+1}Y_{\tilde{\rho}(j)}$, $X=x_1\prod_{j=2}^{N+1}X_{\rho(j)}$ and $\Theta=\prod_{i=1}^{N+1}y_i$, the unitary transformation U_2 may take the form of the following 4×4 matrix

$$U_2=\begin{pmatrix}\frac{\Theta}{Y} & 0 & \sqrt{1-\frac{\Theta^2}{Y^2}} & 0\\ 0 & \frac{\Theta}{X} & 0 & \sqrt{1-\frac{\Theta^2}{X^2}}\\ \sqrt{1-\frac{\Theta^2}{Y^2}} & 0 & -\frac{\Theta}{Y} & 0\\ 0 & \sqrt{1-\frac{\Theta^2}{X^2}} & 0 & -\frac{\Theta}{X}\end{pmatrix}, \tag{31}$$

which turns the state which describe in (30) into

$$\begin{aligned}&U_2U_1|\Delta\rangle_{B_2B_3\ldots B_{N+1}}\otimes|0\rangle_C\\ &\quad=\Gamma\Theta(\alpha|0\cdots0\rangle_{B_2B_3\ldots B_{N+1}}+\beta|1\cdots1\rangle_{B_2B_3\ldots B_{N+1}})\otimes|0\rangle_C\\ &\qquad+\Gamma(\sqrt{Y^2-\Theta^2}|0\cdots0\rangle_{B_2B_3\ldots B_{N+1}}+\sqrt{X^2-\Theta^2}|1\cdots1\rangle_{B_2B_3\ldots B_{N+1}})\otimes|1\rangle_C.\end{aligned} \tag{32}$$

Finally, Bob measures the state of the auxiliary particle C. Obviously, remote preparation of the quantum state is successfully realized if Bob's result is $|0\rangle_C$, and the probability of successful RSP is $F_1^2F_2^2Y^2$. However, if $Alice_1$'s ($Alice_2$) measurement outcome is not $|\varphi_\perp^l\rangle(|\lambda_\perp\rangle)$, the state of particles $B_2, B_3,\ldots,B_{N+1}$ will collapse into the state which describe in (23)–(25). Since Bob has no knowledge of the state $|\Lambda\rangle$, he can't convert it into the original state, that means RSP fails. Thus, the total probability of successful RSP is $2^NF_1^2F_2^2Y^2$. In the whose RSP process, the classical communication cost also can be calculated by using the similar way which discussed detail in Sect. 2.

Our scheme also can be generalized to the multi-parties case. If there are M parties ($Alice_1, Alice_2,\ldots,Alice_M$) who sharing the original state described in (16). Suppose $Alice_1, Alice_1,\ldots,Alice_M$ know partly of the state which they want to help the receiver Bob remotely preparation, that is, $Alice_1$ knows α_1 and β_1, $Alice_2$ knows α_2 and $\beta_2,\ldots,Alice_M$ knows α_M and β_M, where $\alpha=\alpha_1\times\alpha_2\times\cdots\times\alpha_M$ and $\beta=\beta_1\times\beta_2\times\cdots\times\beta_M$. Furthermore, $\alpha_1,\alpha_2,\ldots,\alpha_M$ are real numbers, $\beta_1,\beta_2,\ldots,\beta_M$ are complexes, respectively. The quantum channel shared by M senders and Bob are composed of $(M+N-1)$ paries two-particle non-maximally entangled state ($|\varphi\rangle=x_i|00\rangle_{A_iB_i}+y_i|11\rangle_{A_iB_i}$, $i=1,2,\ldots,(N+M-1)$,

$x_i \geq |y_i|$). We also assume particles $A_1, A_2, \ldots, A_{(N+M-1)}$ belong to $Alice_1$, $Alice_2$ belong to the particle $B_1, \ldots, Alice_{(M)}$ belongs the particle $B_{(M-1)}$, and the sender Bob belongs particles $B_{(M)}, B_{(M+1)}, \ldots, B_{(N+M-1)}$. To remotely prepare state $|\Lambda\rangle$, the senders need to carry out a measurement on their own particles, here, $Alice_1$ should perform $(N+M-1)$-particle projective measurement on his particles, and a vNM should be performed by $Alice_2, Alice_3, \ldots, Alice_M$, respectively. Then, they all inform Bob of their measurement outcome through classical channel. After having received the sender' measurement results, Bob can reconstruct the original state $|\Lambda\rangle$ on his N particles with certain probability by appropriate operation (just like (11) and (31)). Apparently, only and only if $Alice_1$'s ($Alice_2, \ldots, Alice_M$) result is $|\varphi_\perp^l\rangle(|\lambda_\perp\rangle_{B_1}, \ldots, |\lambda_\perp\rangle_{B_{(M-1)}})$, the RSP can successfully be realized, and the success probability is

$$P = F_1^2 F_2^2 \cdots F_M^2 y_1^2 y_2^2 \cdots y_{(N+M-1)}^2 \tag{33}$$

where $F_i = \frac{1}{\sqrt{\alpha_i^2+\beta_i^2}}$ $(i = 1, 2, \ldots, M)$. Similarly, if the senders' measurement result are other outcome, the RSP fails. However, if α and β are both real numbers, and if the $Alice_1$'s ($Alice_2, \ldots, Alice_M$) measurement results are $|\varphi^l\rangle$ or $|\varphi_\perp^l\rangle$ $(|\lambda\rangle_{B_1}$ or $|\lambda_\perp\rangle_{B_1}, \ldots, |\lambda\rangle_{B_{(M-1)}}$ or $|\lambda_\perp\rangle_{B_{(M-1)}})$, Bob can construct the entangled state $|\Lambda\rangle$ by suitable unitary operation and some classical information.

4 Conclusion

To summarize, a scheme for joint remote preparation of the multipartite GHZ-class state is proposed. In this scheme, the sender $Alice_1$ needs to perform N-particle projective measurement and the other senders carry out the vNM on their own particle. After this, they inform the receiver of the measurement results through the classical channel. Thus, the receiver can obtain the original state with certain probability by means of some appropriate unitary operation. Admittedly, multi-particle projective measurement is extremely difficult from a technical point of view in our RSP schemes. However, unlike [24] using multiparticle GHZ entangled as the quantum channel, here we realize RSP by using the non-maximally two-particle entangled channel, which is much easier to produce experimentally than the multiparticle GHZ entangled state. Note that, to remotely preparation multi-particle GHZ state $\alpha|0\cdots 0\rangle + \beta|1\cdots 1\rangle$ in [24] or a JRSP task using a pair of EPR states [25], controlled-NOT (CNOT) operations must be performed by the receiver. This will not take place in our scheme. In principle, as long as $Alice_1$ can make a collective measurement on her own N particles, it will not affect our results of RSP even if the N particles at Bob's place are distantly separate from each other. Furthermore, we find that the success probability of RSP is not only determined by the small coefficients of the quantum channel, but also depended on the coefficients $\alpha_1, \ldots, \alpha_M$ and $\beta_1, \ldots, \beta_M$. In addition, the probability of successful RSP decreasing with the sender's number M increased. Although the success probability will reduce, the scheme's security has been improved. Unlike most of used RSP schemes just contain only one sender, the information of the original state is shared in multi-senders. In our scheme, the receiver must collect all the classical information of the M-senders. Only in this way, can our RSP scheme work. Hence, if one of the M senders finds that the receiver is a dishonest agent, he refuse to publish his measurement result, the RSP fails. Obviously, the more original state senders, the safer of our assignment. Nowadays, using joint measurement [18, 26] or other ways to remote preparation of a class of multi-qubit state (i.e.,

$\alpha|0\cdots0\rangle + \beta|1\cdots1\rangle$) has received much attention [16, 24] due to its important applications in quantum information process. We believe that our scheme will be helpful to realize the essence of the classical communication in a multi-parties RSP process, and it is also useful in expanding RSP field of quantum information.

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